<u>Class notes</u>

Subject : Optics Sem : <u>IV</u> (Major)

Topic : Interference Course : B.Sc. Physics

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Superposition of waves

Principle of superposition of waves : (at a particular point)

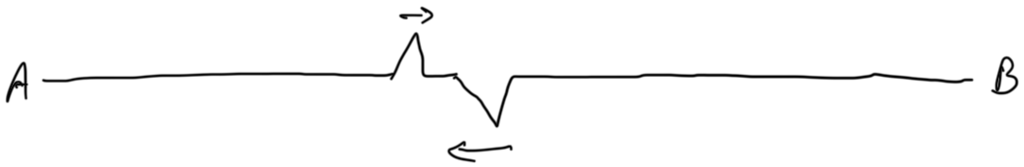
The resultant displacement produced by a number of waves is the vector sum of the displacements produced by each one of the disturbances.

Case I: Let's consider a long string AB. From the end A, a triangular pulse is generated and propagates to the right at speed 'V'.

And from the end B, an identical pulse is generated which start moving toward left with same speed v.

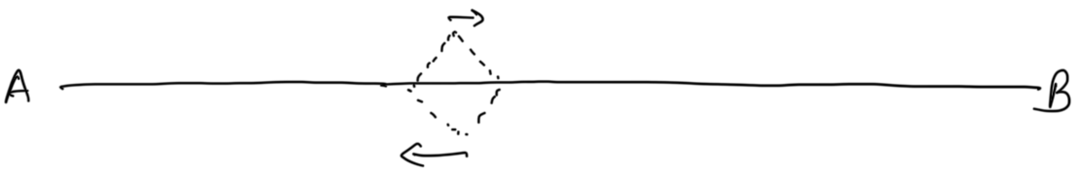
Case II: At a little later time each pulse moves closer to each

Case· other as shown below, but without any interference.



Case: III: At the time of interference, both waves will superpose and the resultant displacement (Solid line, fig. below) will be the algebric addition of each displacement (dashed)

Case IV: Shortly later the two pulses exactly overlap each other and the resultant displacement is zero.

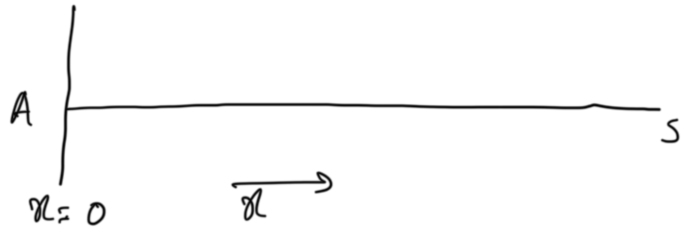


Case V: After some time the impulses cross each other and move further as if nothing had happened.

Here, Case III and IV are characteristic feature of ~~super~~ interference/superposition of waves.

Stationary waves on a string: →

Let's consider a string which is fixed at the point A.



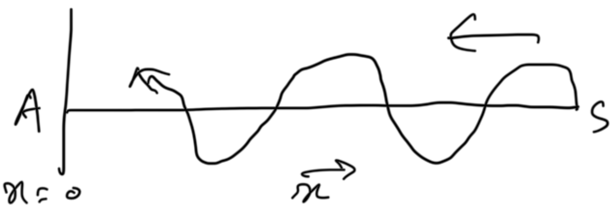
The displacement on the string at any point would be given by :→

$$y_i = a \sin\left[\frac{2\pi}{\lambda}(x + vt) + \phi\right]$$

where, y_i → displacement due to incident wave
a → amplitude of incident wave
x → displacement in x- direction (i.e, position of the wave)
v → velocity of the wave
ϕ → Initial phase

for simplicity, let's consider $\phi = 0$ and the ~~wave~~ incident wave is ~~g~~ generated from point 'S'

i.e,



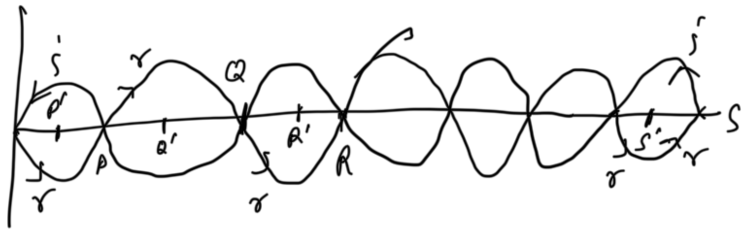
$$y_i = a \sin\left[\frac{2\pi}{\lambda}(x + vt)\right]$$

at point $x = 0$; $y_i = a \sin \frac{2\pi v t}{\lambda}$

let's assume $\frac{v}{\lambda} = \bar{v}$

$$\Rightarrow \quad y_i = a \sin(2\pi \bar{\nu} t)$$

Now, when the wave strikes at ~~A~~ the fixed end A, a reflected wave will be generated and move opposite to incident wave :→



$$y_r \Big|_{x=0} = -a \sin(2\pi \bar{\nu} t)$$

$y_r \Rightarrow$ displacement of reflected wave.

The propagation eq^n of the reflected wave will be

$$y_r = + a \sin\left[2\pi\left(\frac{x}{\lambda} - \bar{\nu} t\right)\right]$$

Hence, the resultant displacement will be :→

$$y = y_i + y_r = a\left[\sin\left\{2\pi\left(\frac{x}{\lambda} + \nu t\right)\right\} + \sin\left\{2\pi\left(\frac{x}{\lambda} - \nu t\right)\right\}\right]$$

$$= 2a \sin\left(\frac{2\pi}{\lambda} x\right) \cos(2\pi \bar{\nu} t)$$

using $\sin(A+B) + \sin(A-B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$\Rightarrow \quad y = 2a \sin\left(\frac{2\pi x}{\lambda}\right) \cos\left(\frac{2\pi v}{\lambda} t\right)$$

Nodes : $\rightarrow$ Such points wher displacement is zero. i.e, $y = 0$

i.e, $\sin \frac{2\pi}{\lambda} x = 0$

$\Rightarrow \quad x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda \ldots$

$$\boxed{x = n\frac{\lambda}{2} \quad \text{wher } n = 0, 1, 2, \ldots}$$

Antinodes : $\rightarrow$ Such points where displacement is maximum :-

$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$

$$\boxed{x = \frac{n\lambda}{4} ; \; n = 1, 3, 5, \ldots}$$

displacement at antinodes : $y = \pm 2a \cos 2\pi \nu t$